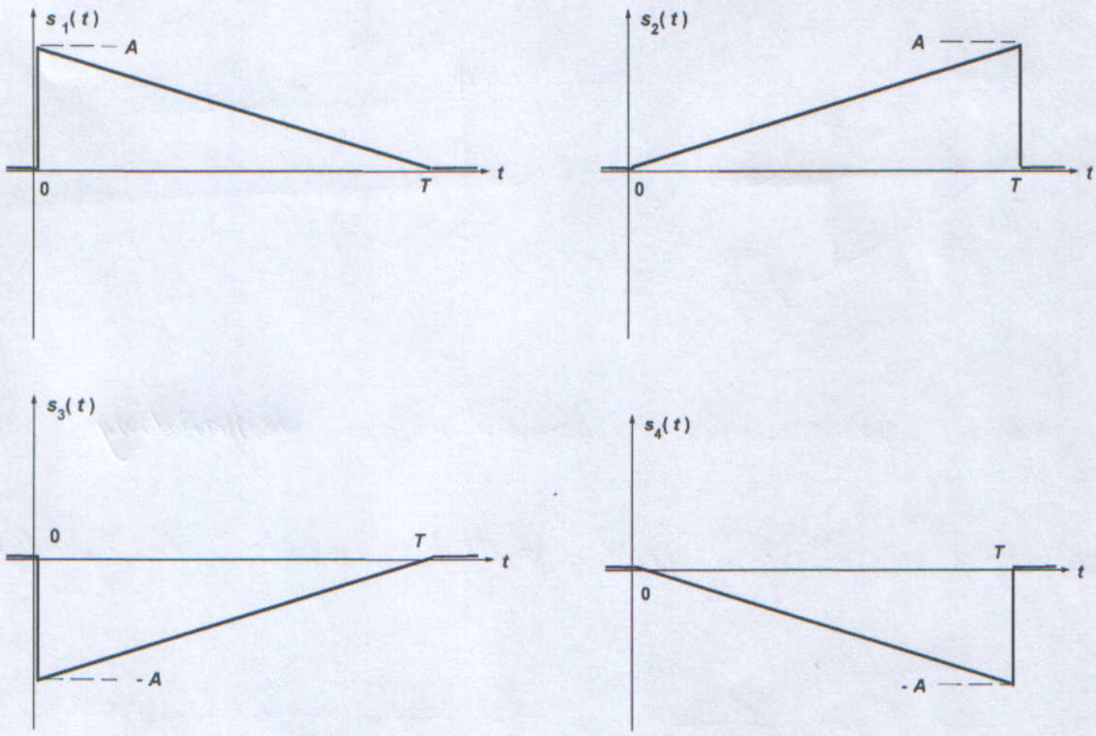
# Çankaya University – ECE Department – ECE 376

Student Name :  
Student Number :

Open source exam  
Duration : 2 hours

Questions

1. (70 Points) The four signals $s_1$(t), $s_2$(t), $s_3$(t) and $s_4$(t) are given as below. Determine what type of modulation (i.e., ASK, PSK, QAM, FSK), these signals represent and the parameters *M* and *N*. Find set of $\psi_i(t)$ **orthogonal** basis functions for $s_1$(t), $s_2$(t), $s_3$(t) and $s_4$(t). Draw the signal constellation diagram, showing the position, the length of vectors $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$ and $\mathbf{s}_4$ and the distance between them. Comment whether the spacings between $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$ and $\mathbf{s}_4$ are optimum (that is equally distributed on the constellation diagram), if not suggest ways to improve it by indicating this optimum arrangement on the constellation diagram. Draw the receiver diagram as correlator and MF, find the output from the MF detector, when $s_1$(t) was transmitted and compare it to the output obtained from the correlator.



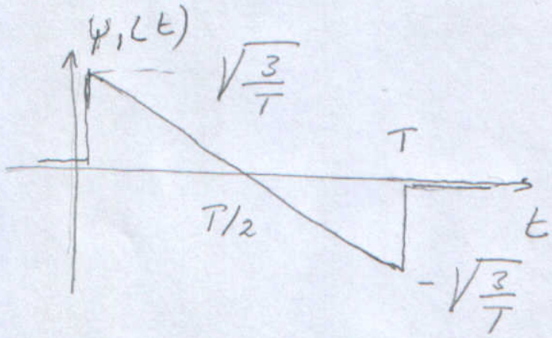

Note : Keep in mind that $\psi_i(t)$ basis functions must satisfy the **orthogonality** condition.

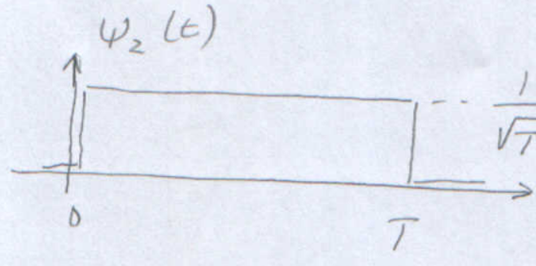
Solution: $s_1(t)$ and $s_2(t)$ can be written as follows;

$$s_1(t) = \frac{A}{T}(T-t) = A\left(1-\frac{t}{T}\right), \quad s_2(t) = \frac{At}{T}$$

$$s_3(t) = -s_1(t), \quad s_4(t) = -s_2(t)$$

Because of the relations of $s_3(t)$ and $s_4(t)$ to $s_1(t)$ and $s_2(t)$, also keeping in mind that $s_1(t)$ and $s_2(t)$ cannot be expressed in terms of each other, so a minimum of two basis functions will be needed. By observing the rule of orthogonality, these may be as follows




The peak values are found by demanding that energies of $\varphi_1(t)$ and $\varphi_2(t)$ equate to unity. Thus

$$\varphi_1(t) = \sqrt{\frac{3}{T}}\left(1 - \frac{2t}{T}\right), \quad \varphi_2(t) = \frac{1}{\sqrt{T}} \qquad 0 < t < T$$

It is easy to see from the above illustrations

$$\int_0^T \varphi_1(t)\,\varphi_2(t)\,dt = 0$$

ELE 376 MT Insert Page for Q 2 - (2)

Signals $s_1(t)$ --- $s_4(t)$ in terms of $\varphi_1(t)$ and $\varphi_2(t)$

$$s_1(t) = \frac{A}{2}\sqrt{\frac{T}{3}}\,\varphi_1(t) + \frac{A}{2}\sqrt{T}\,\varphi_2(t)$$

$$s_2(t) = -\frac{A}{2}\sqrt{\frac{T}{3}}\,\varphi_1(t) + \frac{A}{2}\sqrt{T}\,\varphi_2(t)$$

$$s_3(t) = -\frac{A}{2}\sqrt{\frac{T}{3}}\,\varphi_1(t) - \frac{A}{2}\sqrt{T}\,\varphi_2(t)$$

$$s_4(t) = \frac{A}{2}\sqrt{\frac{T}{3}}\,\varphi_1(t) - \frac{A}{2}\sqrt{T}\,\varphi_2(t)$$

The coefficients in front are the signal vector elements

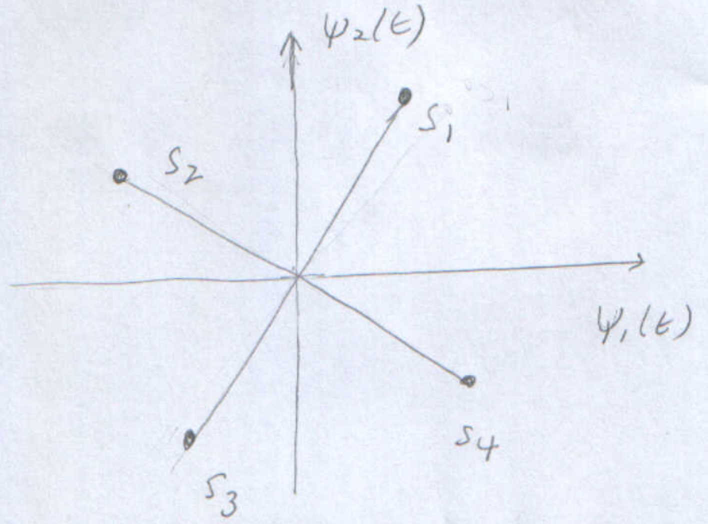
Hence

$$s_1 = \left[\overbrace{\frac{A}{2}\sqrt{\frac{T}{3}}}^{s_{11}},\ \overbrace{\frac{A}{2}\sqrt{T}}^{s_{12}}\right],\quad s_2 = \left[-\frac{A}{2}\sqrt{\frac{T}{3}},\ \frac{A}{2}\sqrt{T}\right]$$

$$s_3 = \left[-\frac{A}{2}\sqrt{\frac{T}{3}},\ -\frac{A}{2}\sqrt{T}\right],\quad s_4 = \left[\frac{A}{2}\sqrt{\frac{T}{3}},\ -\frac{A}{2}\sqrt{T}\right]$$

$$(\text{Length of } s_1 \cdots s_4)^2 = \frac{A^2 T}{3}$$

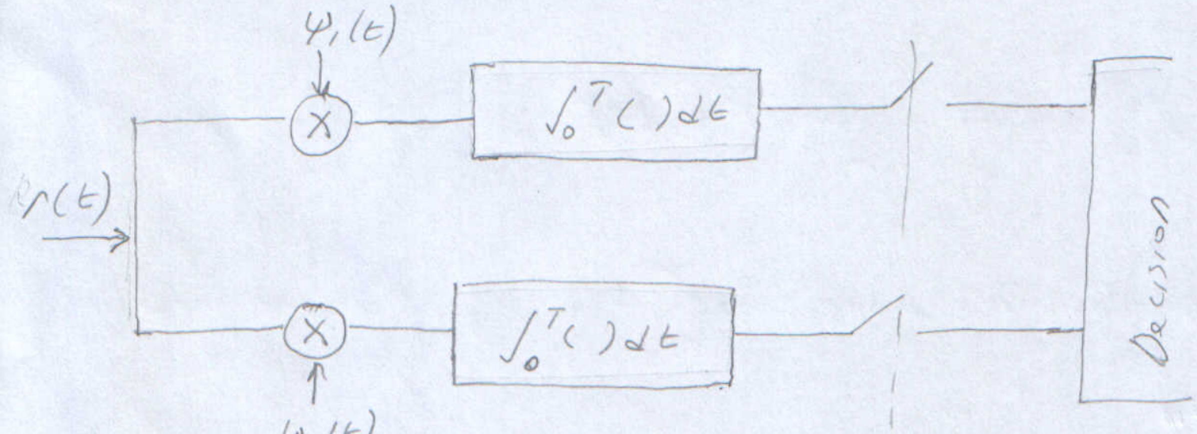
$= \text{Energies of } s_1(t), s_2(t)$
$s_3(t), s_4(t)$




Constellation Diagram

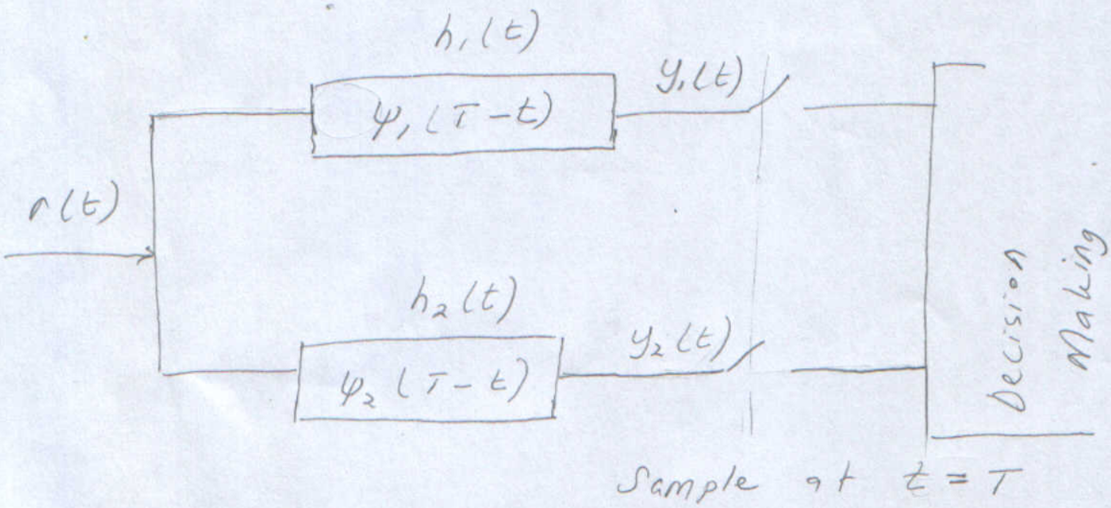
The spacing is optimum, since rotation of $\mp 15°$ will give the well known 4 PSK or 4 QAM.
So optimum, no need for rearrangement

Receiver diagram as correlator




Sample at $t=T$

Receiver diagram as MF (Matched Filter)



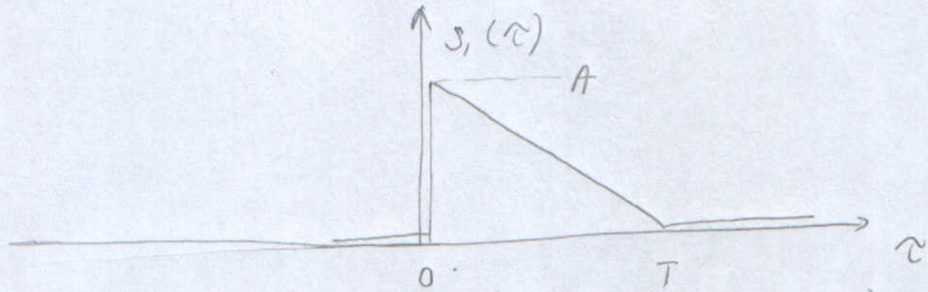

Sample at $t=T$

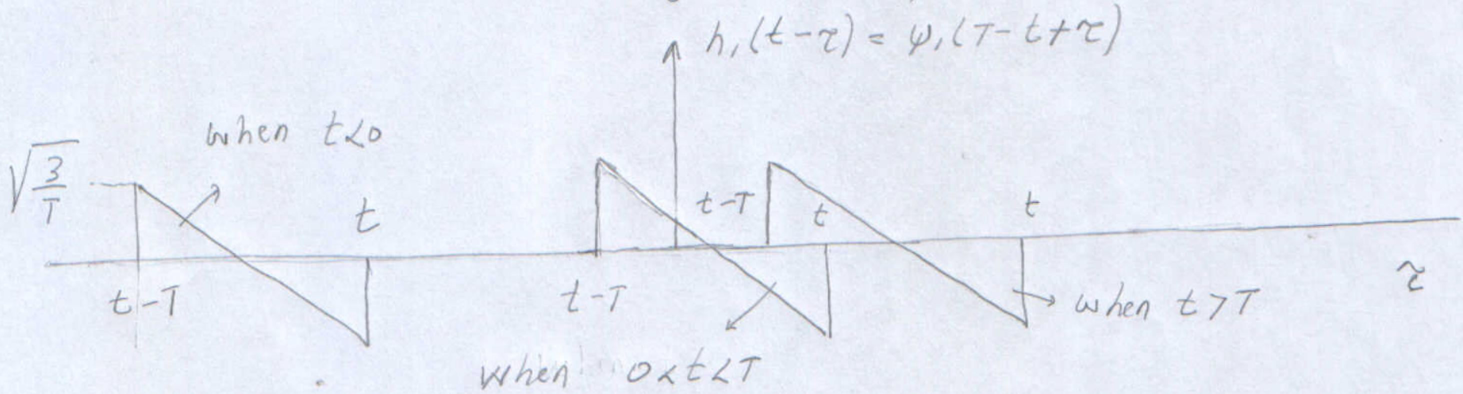
Taking the case of MF and $s_1(t)$ being transmitted, the convolution integrals and convolution graphs would be as follows;

$$y_1(t) = \int_0^T r(\tau)\, h_1(t-\tau)\, d\tau$$

$$= \int_0^T s_1(t)\, h_1(t-\tau)\, d\tau + \int_0^T n(t)\, h_1(t-\tau)\, d\tau$$

$$y_2(t) = \int_0^T r(\tau)\, h_2(t-\tau)\, d\tau$$




(signal part only)

For upper branch of MF receiver at $0<t<T$

begining of overlap

$$y_{1|s_1}(t) = \int_0^t s_1(\tau)\, \varphi_1(T-t+\tau)\, d\tau$$

ECE 376 MT 30.03.2009 – Q1 – Insert Page (4)

$$y_{11s_1}(t) = \frac{A}{T}\sqrt{\frac{3}{T}}\int_0^t \left[T-\tau - \frac{2(T^2 - Tt + \tau t - \tau^2)}{T}\right] d\tau$$

$$= \frac{A}{2}\sqrt{\frac{3}{T}}\left(Tt - \frac{t^2}{2} - \frac{2T^2 t - 2Tt^2 + t^3 - 2t^3/3}{T}\right)$$

More detailed steps are in EEM 467 MT2 Solutions

dated 11.12.2007.

Tests : $y_{11s_1}(t=0) = 0$ , $y_{11s_1}(t=T) = \overbrace{\frac{A}{2}\sqrt{\frac{T}{3}}}^{s_{11} \text{ in } s_1}$ : OK

At $T < t < 2T \rightarrow$ end of overlap

$$y_{12s_1}(t) = \int_{t-T}^{T} s_1(\tau)\,\varphi_1(T - t + \tau)\,d\tau$$

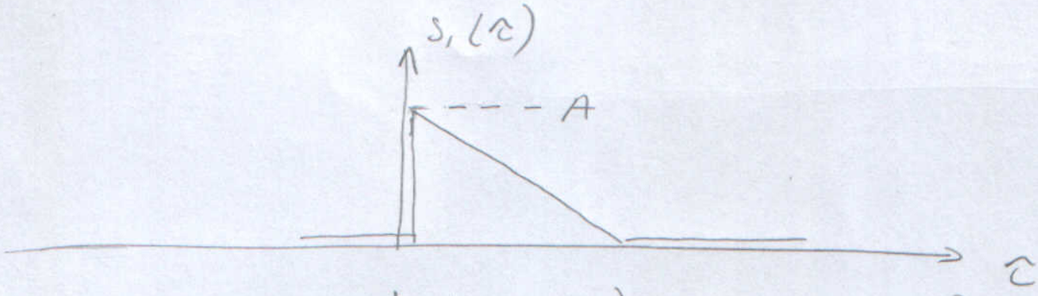
$$= \frac{A}{2}\sqrt{\frac{3}{T}}\left\{\frac{T^2}{2} - \frac{2(T^3 - T^2 t + 0.5T^2 t - T^3/3)}{T}\right.$$

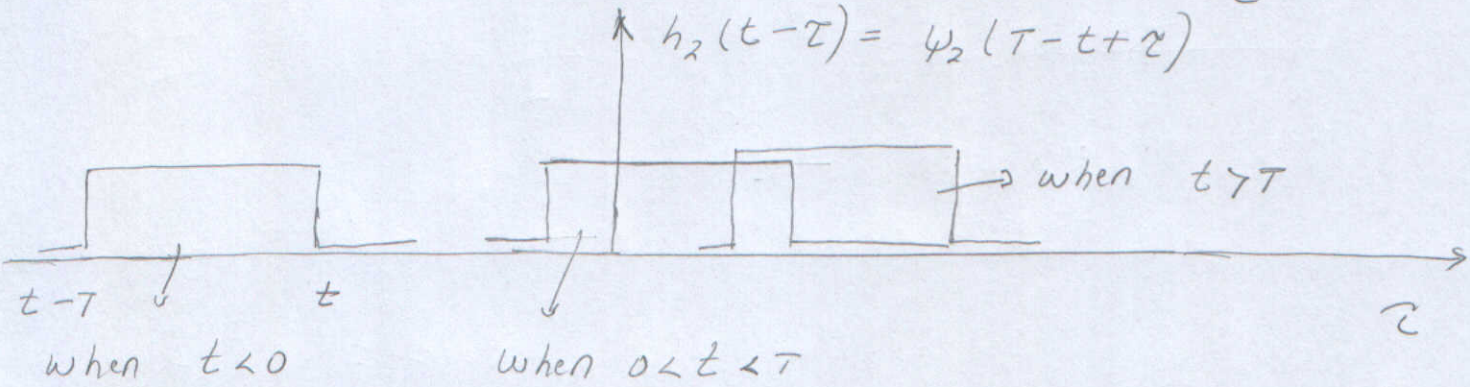
$$\left. - T(t-T) + \frac{(t-T)^2}{2} + \frac{2[T^2(t-T) - Tt(t-T) + 0.5t(t-T)^2 - (t-T)^3/3]}{T}\right\}$$

Tests $y_{12s_1}(t=T) = \frac{A}{2}\sqrt{\frac{T}{3}}$ , $y_{12s_1}(t=2T) = 0$ ; OK

The OPs of MF for the lower branch is

shown overleaf




At $0<t<T$

$$y_{21s_1}(t) = \int_0^t s_1(\tau)\, h_2(t-\tau)\, d\tau$$

$$= \frac{A}{T\sqrt{T}} \left( Tt - \frac{t^2}{2} \right)$$

Tests $y_{21s_1}(t=0) = 0$ , $y_{21s_1}(t=T) = \overbrace{\frac{A\sqrt{T}}{2}}^{s_{12}}$

At $T<t<2T$

$$y_{22s_1}(t) = \int_{t-T}^{T} s_1(\tau)\, h_2(t-\tau)\, d\tau$$

$$= \frac{A}{T\sqrt{T}} \left[ \frac{T^2}{2} - T(t-T) + \frac{(t-T)^2}{2} \right]$$

At $T < t < 2T$

$$y_{22s_1}(t) = \int_{t-T}^{T} s_1(\tau)\, h_2(t-\tau)\, d\tau$$

$$= \frac{A}{T\sqrt{T}} \left[ \frac{T^2}{2} - T(t-T) + \frac{(t-T)^2}{2} \right]$$

Tests $\quad y_{22s_1}(t=T) = \dfrac{A\sqrt{T}}{2}$ , $y_{22s_1}(t=2T) = 0$ ; OK

The correlator will give the OP after integration (not $t$ dependent), it should match the above at $t = T$.

$$y^{c}_{1s_1}(t) = \int_0^T s_1(t)\, \varphi_1(t)\, dt$$

$$= \frac{A}{T}\sqrt{\frac{3}{T}} \int_0^T \left(T - 3t + \frac{2t^2}{T}\right) dt$$

$$= \frac{A}{T}\sqrt{\frac{3}{T}} \left(T^2 - \frac{3T^2}{2} + \frac{2T^2}{3}\right) = \frac{A}{2}\sqrt{\frac{T}{3}}$$

Same as $y_{11s_1}(t=T) = y^{MF}_{12s_1}(t=T)$ of MF

$$\hat{y}_{12s_1}(t) = \int_0^T s_1(t)\, \varphi_2(t)\, dt$$

$$= \frac{A}{T} \frac{1}{\sqrt{T}} \int_0^T (T-t)\, dt$$

$$= \frac{A}{T^{3/2}} \Big| Tt - \frac{t^2}{2} \Big|_0^T = \frac{A\sqrt{T}}{2}$$

same as $y_{21s_1}(t=T) = y_{22s_1}(t=T)$

2. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) For demodulation of an FM signal, we use PLL : True, PLL by extracting the phase term in the modulated signal and differentiating this phase obtains the modulating (message) signal

b) Orthogonal signals, when placed on the constellation diagram will have an angle of $180^0$ between them : False, orthogonal signals have an angle of $90°$ between them, when shown on the constellation diagram

c) FSK and PPM are multidimensional signals, establishing orthogonality by dividing the time domain into non-overlapping slots : Partially true, this statement applies to PPM, whereas FSK establishes orthogonality by slicing the frequency axis and using the point of orthogonality between $\Delta f$ and $T$

d) 8 PSK and 8 QAM will need the same channel bandwidth : True, since BW is related purely to the dimensionality of signal space (provided both use the same waveform shapes)

e) A channel with a rectangular frequency response will have a delta function type time response : False, an unlimited channel, i.e. $C(f) = 1$, will have $\delta(t)$ type of time response, a channel with rectangular frequency response will have a sinc type time response